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RtI Math and Number Sense

What Interventions Should You Consider?

We can figure this out

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National Council of Teachers of Mathematics

New Orleans Regional Conference
National Council of Teachers of Mathematics
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RtI

- Response to Intervention (RtI) can be thought of as an early detection, prevention, and ongoing support system that identifies students and provides them with the support they need **before** they fall behind and before they are formally identified and designated for special education services.

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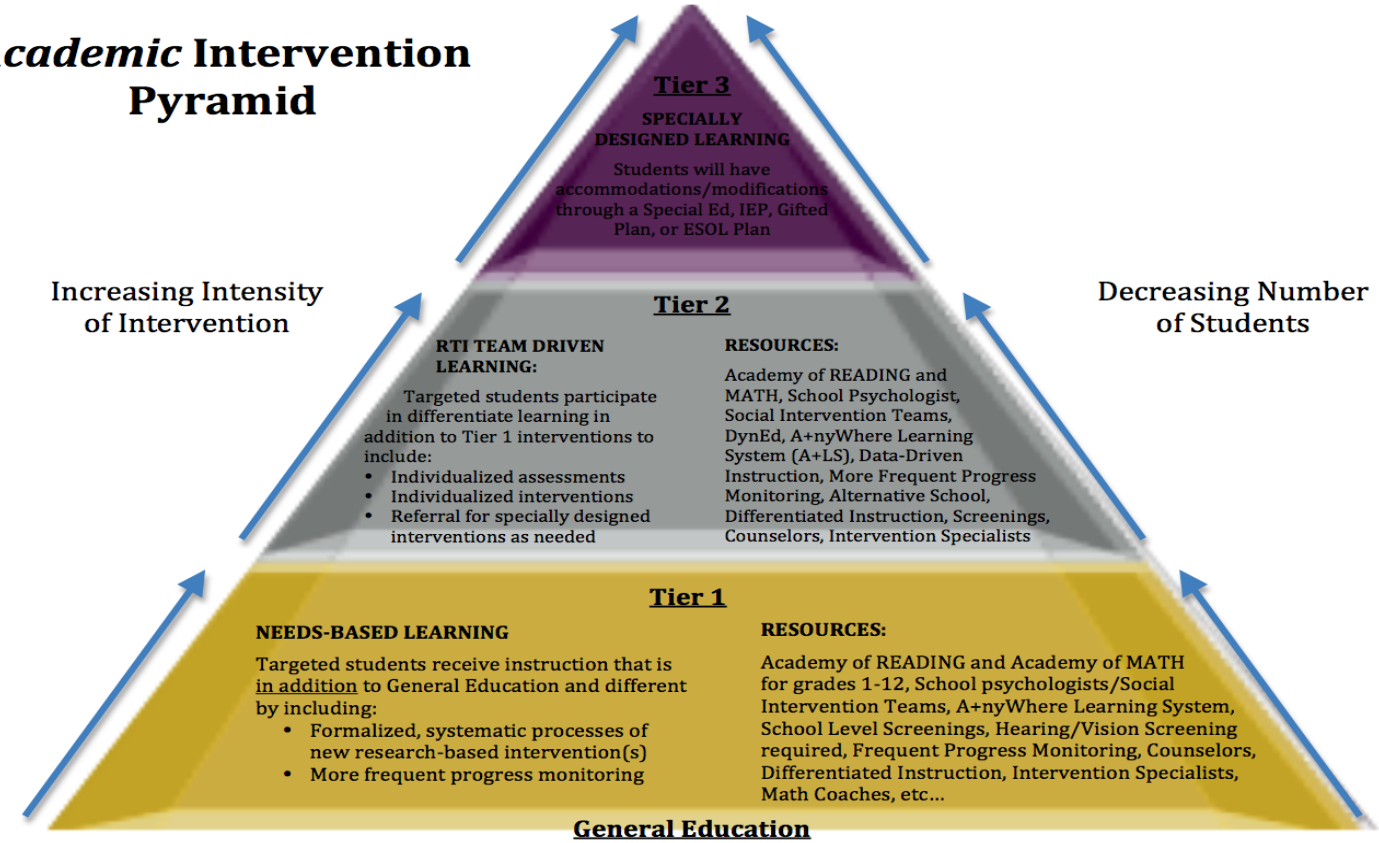
Academic Intervention Pyramid



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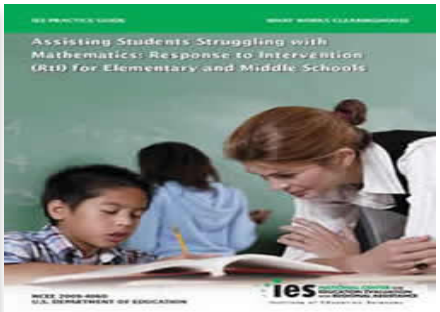
RtI Challenges

- Providing Opportunities in Mathematics.
- Defining the Tiers.
- NOT thinking of RtI as a deficit model!
- RtI as a “lifeline.”



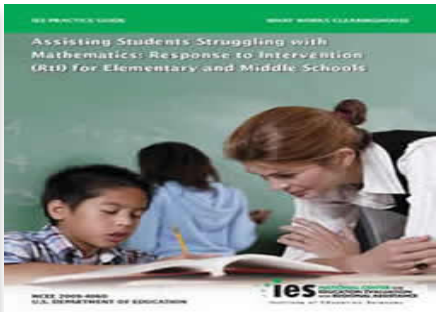
Recommendations 1-4

1. **Screen all students to identify those at risk** for potential mathematics difficulties and provide interventions to students identified as at risk.

2. Instructional materials for students receiving interventions should **focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8**.

3. Instruction during the intervention should be **explicit and systematic**. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

4. Interventions should include instruction on **solving word problems** that is based on common underlying structures.



Recommendations 5-8

5. Intervention materials should include opportunities for students to **work with visual representations of mathematical ideas** and interventionists should be proficient in the use of visual representations of mathematical ideas.

6. Interventions at all grade levels should devote about 10 minutes in each session to building **fluent retrieval of basic arithmetic facts**.

7. **Monitor the progress of students** receiving supplemental instruction and other students who are at risk.

8. **Include motivational strategies** in tier 2 and tier 3 interventions.

IES Practice Guide – RtI and Mathematics – on line and free!!!

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They're all yours…

- Start before they have the instructions.
- Seemingly get distracted by movement – of any type!
- Oh, yeah, I get it!
- I'm done!

Fennell; The Done 1st Generation!

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Adapting Instruction

- Scaffolding – teacher supports provided
- Time needed to learn – more!
- Time students will stay on task – less!
- Homework - important

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Intervention and Remediation

- Some consider both terms synonymous, use them interchangeably, BUT…
- **Intervention**: Plan of action implemented by an instructor on behalf of students who may need extra help or acceleration
  - Students' difficulties or strengths are in *early stages*
  - Intervention ideally addresses weaknesses or strengths *before* they become a problem for the student
- **Remediation**: Actions taken to reverse established patterns of achievement by students who are *already* struggling or failing and who need intensive, long-term help
  - Supplemental instruction on content students should have mastered but have not

Visit NCTM Intervention report, materials, coming work…

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Intervention: Many students benefit

- Struggling students (typically)
- Absentees
- Mathematically talented students, as a challenging supplement to a standard instructional program

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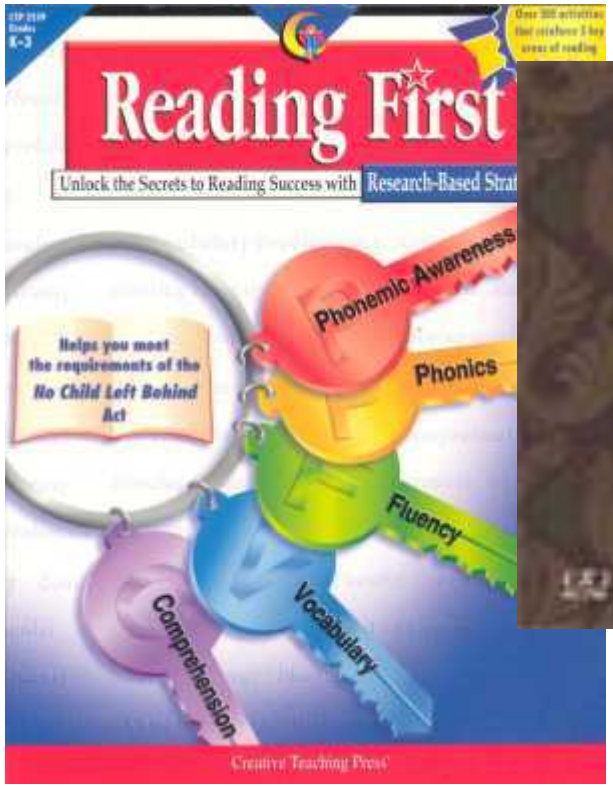
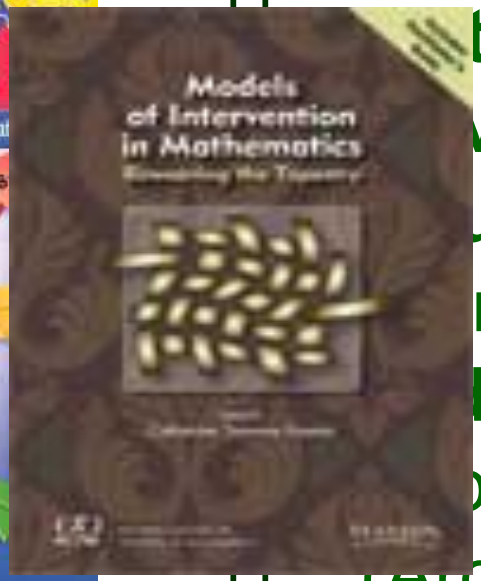
Another Challenge

- 



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ruary 2006
(Archived)

Lot of $

Lot of talk

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Rtl – A window

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Principles and Standards for School Mathematics

Content Standards

- **Number and Operations**
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

Remember? Number Sense

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Domains

- 3, 4, 5
  - **Operations and Algebraic Thinking**
  - **Number and Operations in Base Ten**
  - ***Number and Operations - Fractions***
  - Measurement and Data
  - Geometry

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Grade 4

- **Operations and Algebraic Thinking**
  - Use the four operations with whole numbers to solve problems
  - Gain familiarity with factors and multiples
  - Generate and analyze patterns
- **Number and Operations in Base Ten**
  - Generalize place value understanding for multi-digit whole numbers
  - Use place value understanding and properties of operations to perform multi-digit arithmetic using the standard algorithm for addition and subtraction

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Grade 4 (cont.)

- **Number and Operations - Fractions**
  - Extend understanding of fraction equivalence and ordering.
  - Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
  - Understand decimal notation for fractions and compare decimal fractions.

CCSS – Think About

- Add and subtract within 1000, using concrete models or drawings and *strategies based on place value, properties of operations, and/or the relationship between addition and subtraction*; relate the strategy to a written method…(grade 3)
- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using *strategies based on place value, the properties of operations, and/or the relationship between multiplication and division,* illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

NOTE: Conceptual understanding is not an option, it's an expectation!

Standards for Mathematical Practice

Mathematical Practices	*NCTM Processes*
Make sense of problems and persevere in solving them.	Problem Solving
Reason abstractly and quantitatively.	Reasoning and Proof
Construct viable arguments and critique the reasoning of others.	Reasoning and Proof, Communication
Model with mathematics.	Connections
Use appropriate tools strategically.	Representation
Attend to precision.	Communication
Look for and make use of structure.	Communication, Representation
Look for and express regularity in repeated reasoning.	Reasoning and Proof

Understanding as points of intersection between expectations and practices

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The Focus Here

- *Recognizing understanding as points of intersection between CCSS expectations (standards) and the mathematical practices.*
  - *A sense of number, place value, etc.*
  - *Basic Facts*
  - *Multiplication and Division*
  - *Fractions*

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1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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On each numbered 100 chart, shade in the numbers that are...

- 2-digit numbers
- numbers containing the digit 3
- numbers < 60
- odd numbers
- numbers between 31 and 51
- numbers where the sum of the digits = 8

WHAT NUMBER DID YOU COLOR ON EVERY CHART?

Marcy Cook, 1992

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On each numbered 100 chart, shade in the numbers that are...

- > 30
- numbers with an odd digit in the tens place
- even numbers
- numbers with 2 as a digit
- numbers between 45 and 95
- numbers with the sum of the digits = 11

WHAT NUMBER DID YOU COLOR ON EVERY CHART?

Marcy Cook, 1992

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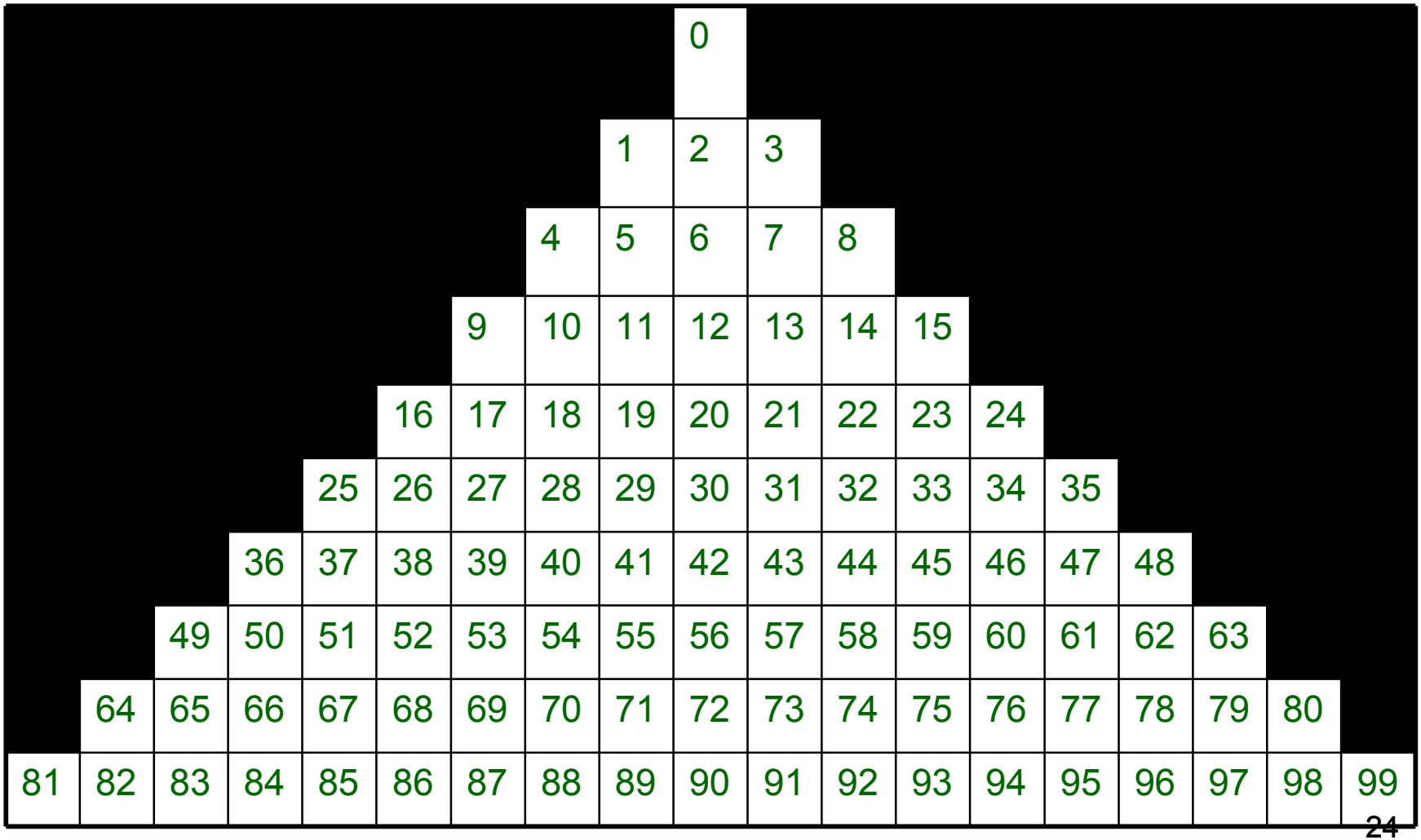
And how about this chart?



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10	20	30	40	50	60	70	80	90	100
110	120	130	140	150	160	170	180	190	200
210	220	230	240	250	260	270	280	290	300
310	320	330	340	350	360	370	380	390	400
410	420	430	440	450	460	470	480	490	500
510	520	530	540	550	560	570	580	590	600
610	620	630	640	650	660	670	680	690	700
710	720	730	740	750	760	770	780	790	800
810	820	830	840	850	860	870	880	890	900
910	920	930	940	950	960	970	980	990	1000

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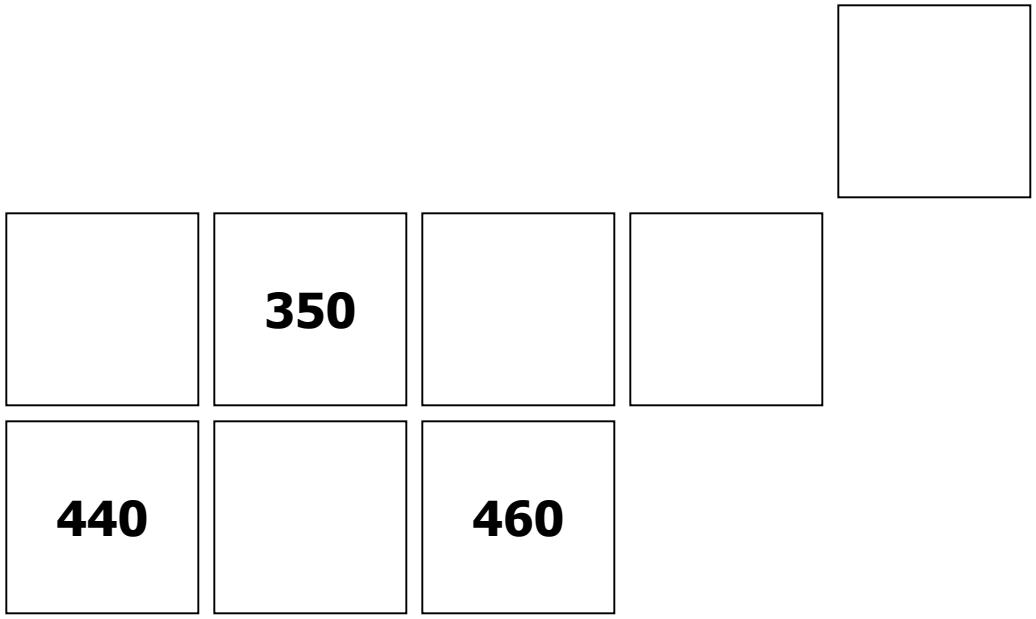
100+ Chart Puzzles



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True or False - 818

- Number of students in your school?
- Number of people in your town?
- Number of players on the team?
- Number of pennies in a collection?
- Closer to 500 or 1,000?
- > 500
- > 750

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0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0

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Think about…

- How many of the decimals are < 1?
- Name a mixed number between 2.5 and 2.7 and indicate where you would place it on the Decimal chart.
- What % of the decimals on the chart are greater than 1?

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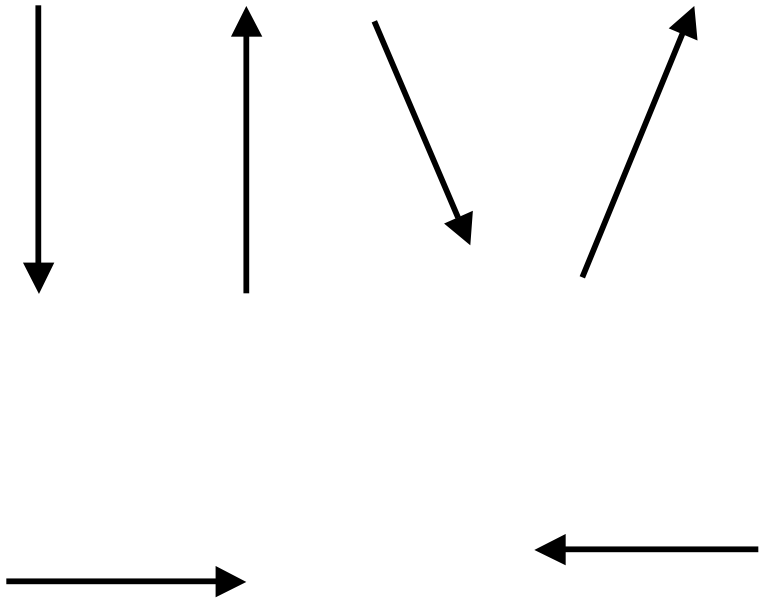
Think about…

- Arrow mental math… (start with 7.7)



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Whole Number - Benchmarks

- 10 – before you
- 100 – before you
  - 100 is a big number when it's:
  - 100 is a small number when it's:

- 1,000
- 1,000,000

Note the change…

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Math Wall Activities

24

73

49

¾

2%

550

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Name something that helps you attach meaning to each number below:

- 25
- 50
- 500
- 75
- 60
- 36
- 30

Note: typical references to time, money,
And measurement…

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Favorites

- Write **3** numbers that have some significance to your life.
- Exchange lists. Provide random clues for the numbers.
- Guess which numbers fit the clues.

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Today's Target is 36

- Try to make today's target by:
  - Adding 2 numbers
  - Finding the difference of 2 numbers
  - Multiplying 2 numbers
  - Adding 3 numbers
  - Multiplying 3 numbers
  - Multiplying and subtracting
  - YOUR own method!

McIntosh, Reys, Reys, and Hope (1997)

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Bar and strip diagrams – proportion implied

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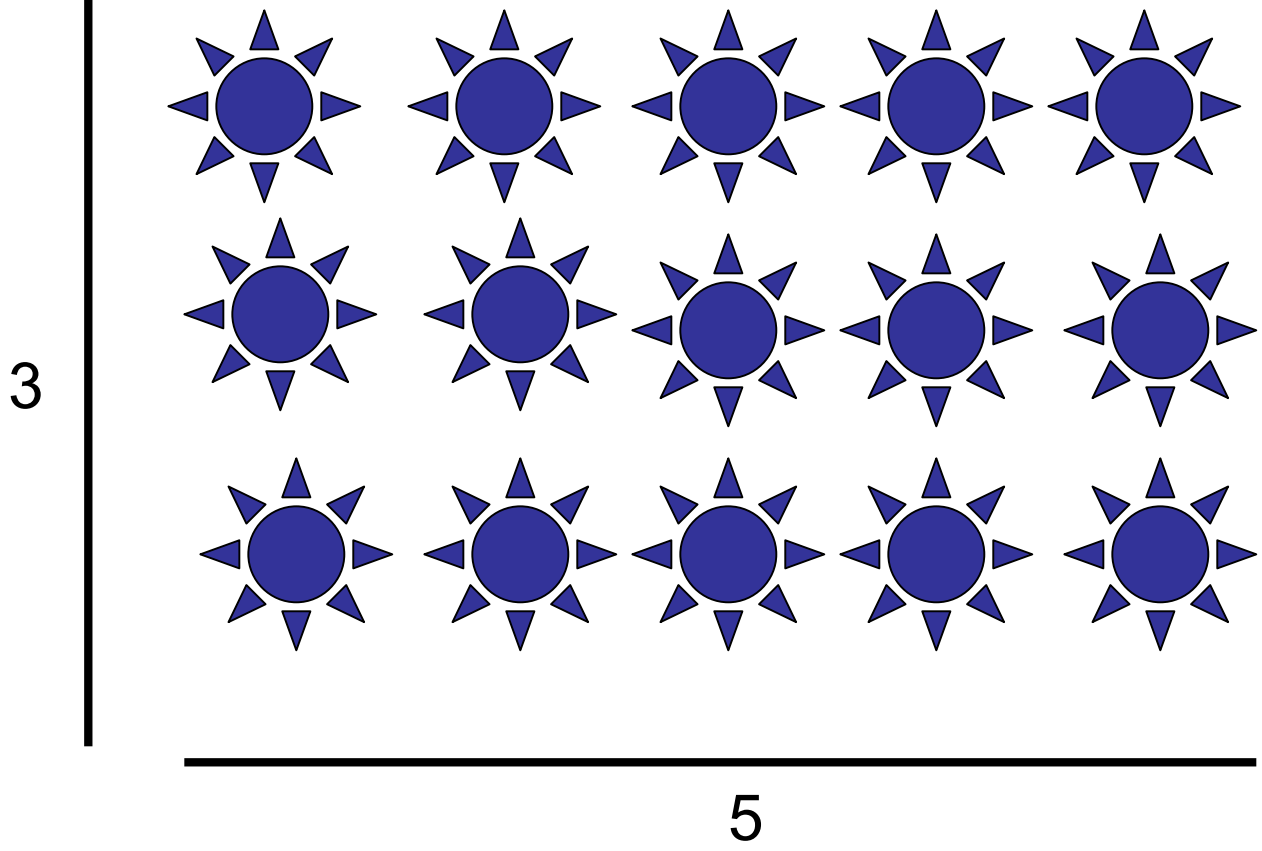

3 by 5 or 5 by 3 array

3 x 5 = 5 x 3 = 15

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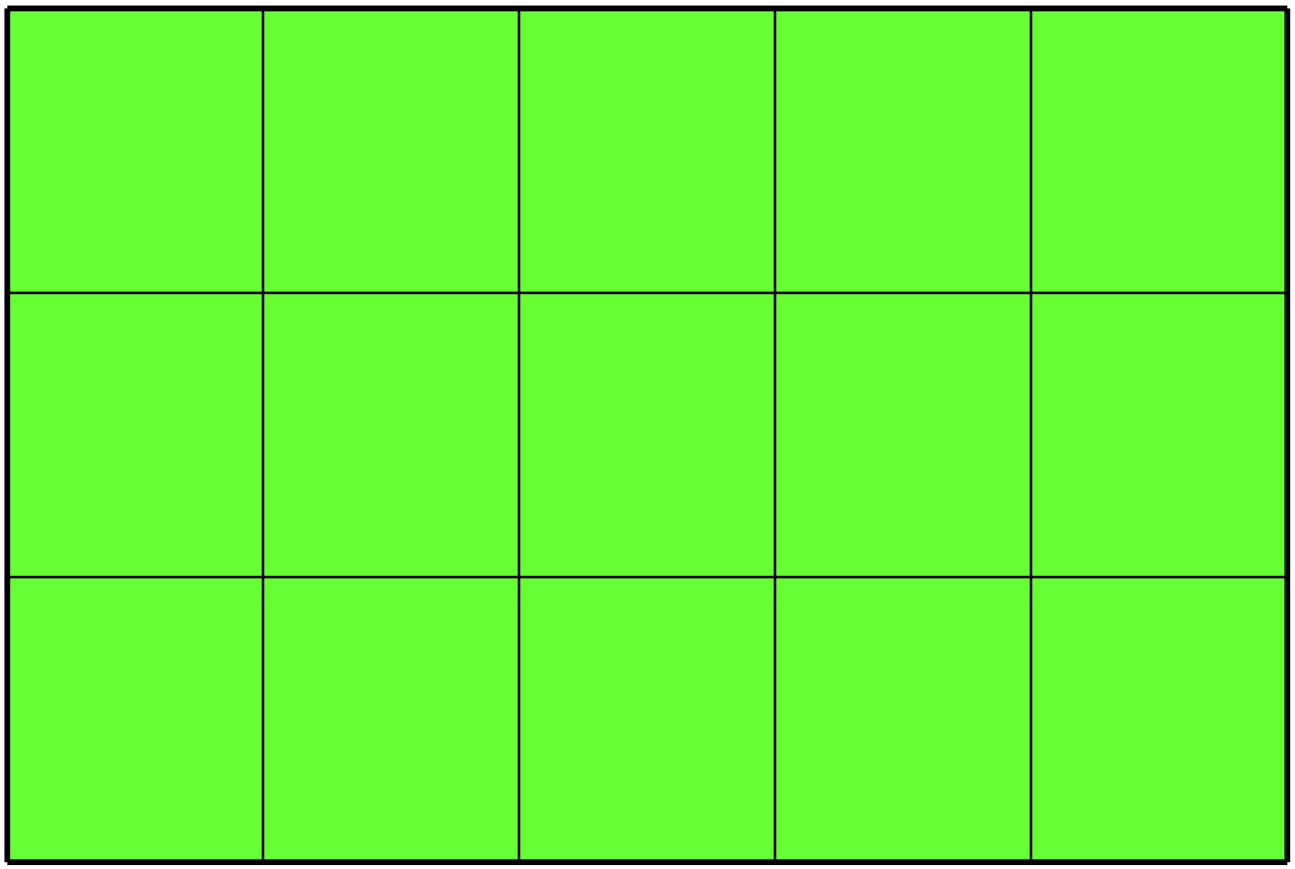
3 by 5 area grid

3 x 5 = 5 x 3 = 15



3

5

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Fact Strategies - Work

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

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Facts – A Sequence?

- 2's – skip counting, doubling, pairs
- 10's – skip counting, base ten blocks, …
- 5's – nickel facts, skip counting, half tens…
- 1's – by itself, identity, be careful…
- 0's – remembering the product is constant

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Facts – and then…

- 3's – tripling, one more group than 2's
- 4's – doubling a double
- 6's – doubling a 3's fact
- 9's – use tens (one less group), tricks
- 8's – doubling 4's
- 7's – use distributive property

- In all cases USE: commutative property and think division (7 x 6 = ?, think ? ÷ 7 = 6)

O'Connell and SanGiovanni, Heineman, in press

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Relate to Money

Nickels	¢
2	
?	15
4	
5	
6	

Dimes	¢
2	
3	
4	
?	50
?	60

Add to this, extend it, etc.

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- Finding and using patterns and other thinking strategies greatly simplifies the task of learning multiplication tables.

Thornton, 1978

- Children need to identify individual products rapidly. **Little is known** about how children acquire this fluency or what experiences might be of most help.

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Multiplication Concepts: Show me…

- Six boxes with 7 hats in each box. How many hats?
  - What's the multiplication fact?
  - Make a picture (sets)
  - Show this as a repeated addition equation
  - Make an array and/or area model
  - Show this on a number line

Interview idea

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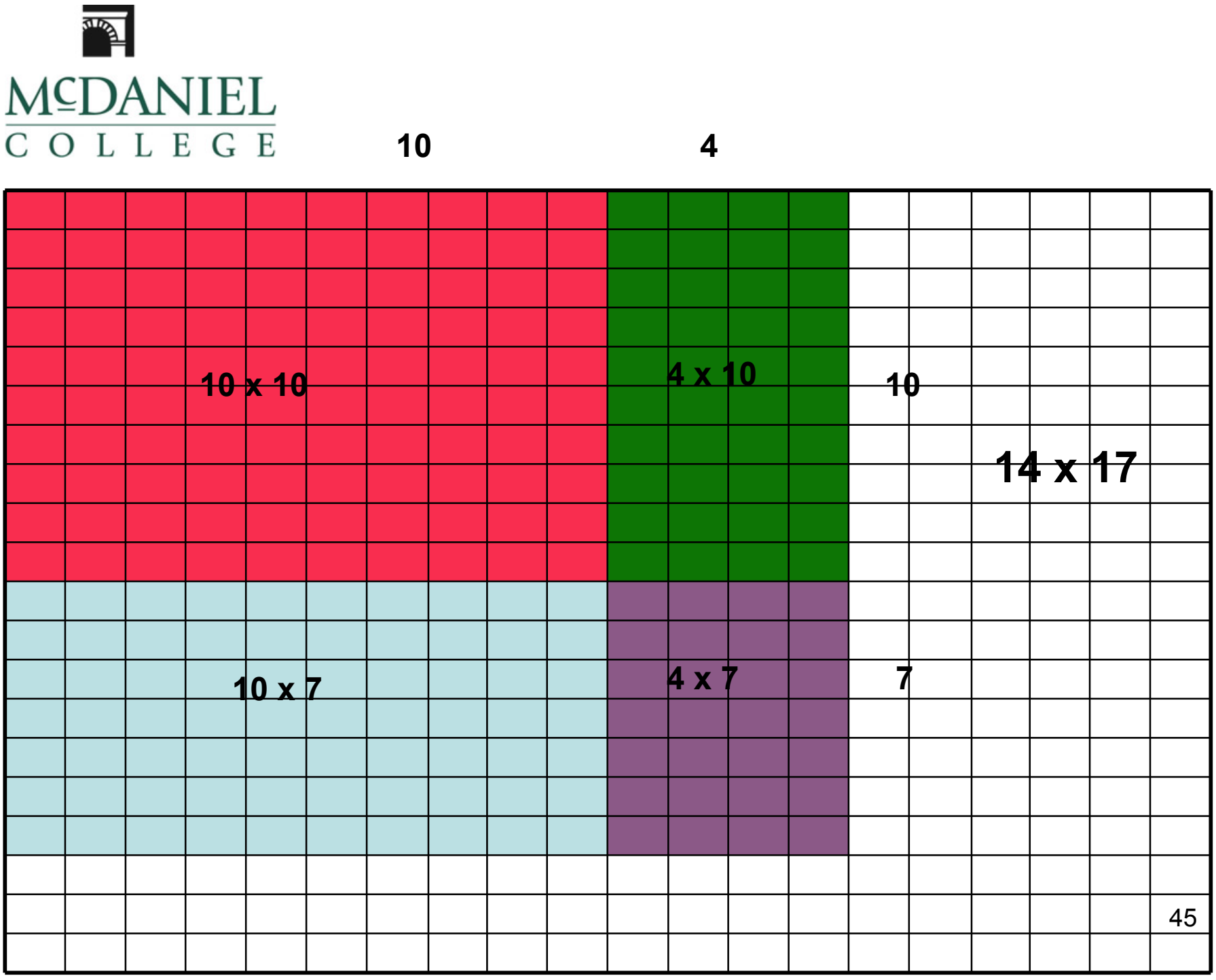

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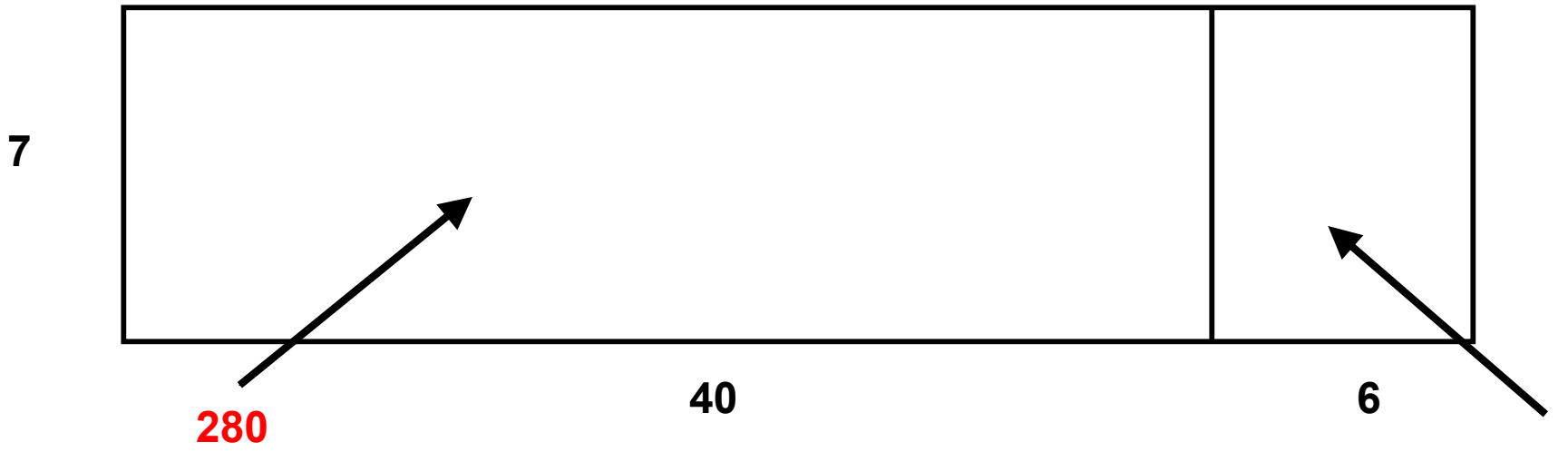
Boxes to multiply…

- Draw a rectangle to show 46 x 7 = 322



46 x 7 = (40 x 7) + (6 x 7)
=

280 + 42 = 322

Navigations 3-5, Number and Operations, 2007

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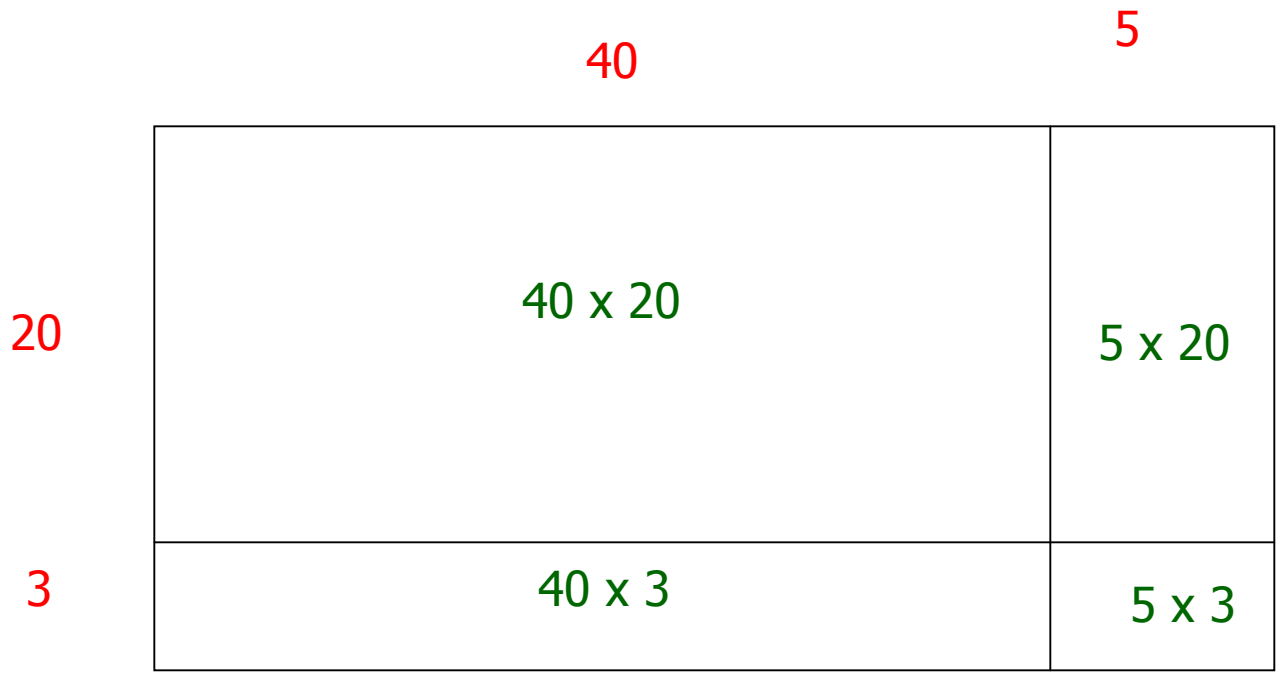
- How about 45 x 23



45
X 23

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Division Concepts

- Measurement – Successive subtraction
  - 37 ÷ 5 = 37 – 5 – 5 – 5 – 5 – 5 – 5 – 5 = 2

    7 5's are subtracted, with 2 left over

  - How many groups?
- Partition – sharing
  - 37 ÷ 5 = 5 r 2, if I share 37 among 5 people
  - How many in each group?

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Division Concepts (cont.)

- Missing Factor Approach

  $7 \times ? = 96;\ 96 \div 7 = ?$

  $125 = 5 \times$ 

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Division - Models

- Division as successive subtraction
  - Nelson had 125 pieces of candy. He gave 7 pieces to each of his friends. How many friends received candy? (how many groups)
- Division as sharing
  - Nelson had 125 pieces of candy. He shared his candy [equally] among his 7 friends. How many pieces of candy did each friend receive? (how many in each group)

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Estimation – Some Thoughts

- Estimating Magnitude – should begin early and occur often.
- Children are initially uncomfortable with computational estimation.
- The language of computational estimation is adult language. Children seem OK with such language as they grow – experientially.

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Estimation

- How many 1-digit numbers are there? 2-digit numbers? 3-digit numbers?
- The toll road is 243 miles long. If you traveled at a speed of 61 mph, about how many hours will you be on the toll road?
- The height of full grown human is about 21 times the length of the middle finger. ARE YOU KIDDIN' ME!!!!!!!

Elementary School Mathematics, Addison-Wesley, 1968

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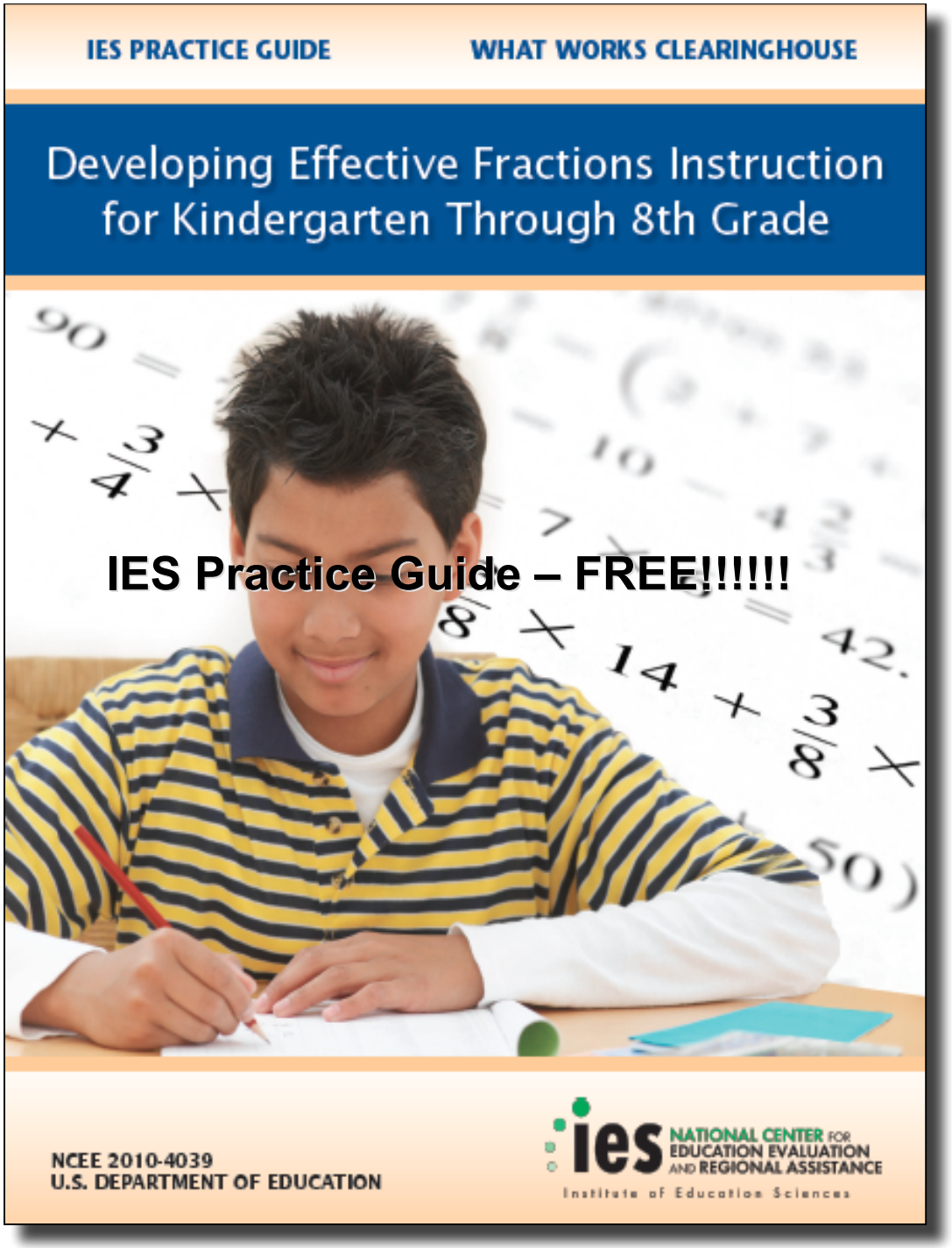
Oh my…



Panel

Robert Siegler (Chair)
Carnegie Mellon University

Thomas Carpenter
University of Wisconsin-Madison

Francis (Skip) Fennell
McDaniel College

David Geary
University of Missouri at Columbia

James Lewis
University of Nebraska-Lincoln

Yukari Okamoto
University of California-Santa Barbara

Laurie Thompson
Elementary Teacher

Jonathan (Jon) Wray
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Staff

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Andrew Gothro
Sarah Prenovitz
Mathematical Policy Research

Project Officer - **Susan Sanchez**
Institution of Education Sciences (IES)

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Recommendation 1

Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.

- Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.
- Extend equal-sharing activities to develop students' understanding of ordering and equivalence of fractions.
- Build on students' informal understanding to develop more advanced understanding of proportional reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

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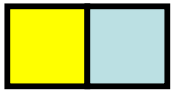
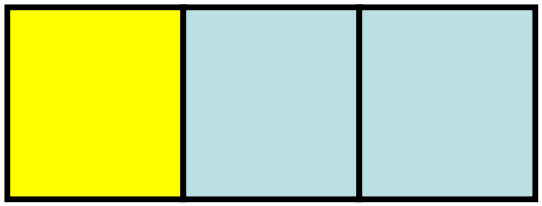
Fraction beginnings…

- which one is larger, 1/2 or 1/3?





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Fraction Sorting

- Sort the fractions below as near: 0, ½, or 1

4/7	1/7	8/9	3/5
2/3	1/10	4/8	6/11
4/5	2/12	9/12	5/12
1/8	3/8	4/9	7/14

- What's alike about all fractions near 1? Near 0?

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- How can we share **eleven** hoagies (aka subs) among four people?
- How can we share **eleven** hoagies (aka subs) among five people?

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How about if we have six people and we need to share 5 cookies?*

Division involving equal shares is a process that many understand intuitively.

*food seems to work – a lot!

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Recommendation 2

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.
- Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms.

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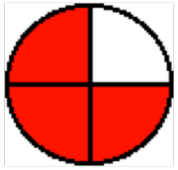
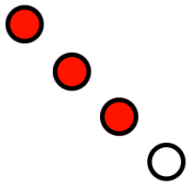
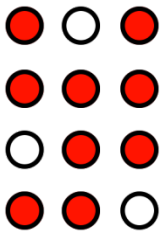
Thinking about ¾…

a) 

b) 

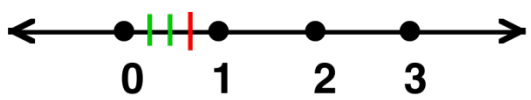
c) 

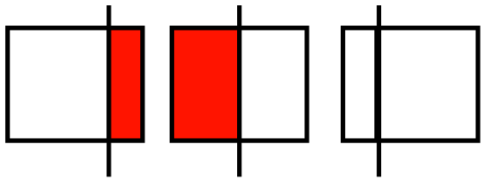
d) How many 4's are there in 3?

e) 18 crayons out of a box of 24

f) .75

g) I want to share 3 bottles of soda equally among 4 people. How much will each person get?

h) 

i) 

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$\frac{3}{4}$

- What happens to the value of the fraction if the numerator is increased by 1?
- What happens to the value of the fraction if the denominator is decreased by 1?
- What happens to the value of the fraction if the denominator is increased?

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Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

- 5/3 5/6 5/5 5/4 5/8
- 7/8 2/8 10/8 3/8 1/8

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You can't make this stuff up!

- The weather reporter on WCRB (a Boston radio station) said there was a 30% chance of rain. The host of the show asked what that meant. The weather reporter said ``It will rain on 30% of the state." ``What are the chances of getting wet if you are in that 30% of the state?" ``100%."

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Recommendation 3

Help students understand why procedures for computations with fractions makes sense.

- Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures.
- *Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.*
- *Address common misconceptions regarding computational procedures with fractions.*
- Present real-world contexts with plausible numbers for problems that involve computing with fractions.

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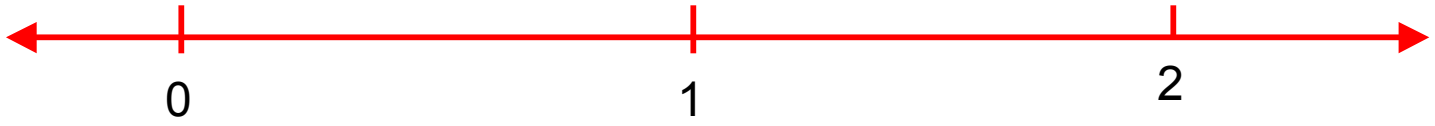
- Tell me about where 2/3 + 1/6 would be on this number line (Cramer, Henry, 2002).



Sense Making:

“2/3 is almost 1, 1/6 is a bit more, but the sum is < 1”

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7/8 – 1/8 = ?

- Interviewer: Melanie these two circles represent pies that were each cut into eight pieces for a party. This pie on the left had seven pieces eaten from it. How much pie is left there?
- **Melanie:** ***One-eighth, writes 1/8*****.**
- Interviewer: The pie on the right had three pieces eaten from it. How much is left of that pie?
- **Melanie:** ***Five-eighths, writes 5/8.***
- Interviewer: If you put those two together, how much of a pie is left?
- **Melanie:** ***Six-eighths, writes 6/8.***
- Interviewer: Could you write a number sentence to show what you just did?
- **Melanie:** ***Writes 1/8 + 5/8 = 6/16.***
- Interviewer: That's not the same as you told me before. Is that OK?
- **Melanie:** ***Yes, this is the answer you get when you add fractions.***

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What Happens Here?

- 1/2 x 3/4 **< or >** 3/4
- 3/4 x 1/2 **< or >** 1/2

- 1/2 ÷ 3/4 **< or >** 1/2
- 3/4 ÷ 1/2 **< or >** 3/4

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Now what?

- There are 25 students in our class. Each student will get ¼ of a pizza. Your job is to find out how many pizzas we should order. Be sure to show your work.

- How many pizzas should we order?

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Lakers vs Nuggets

- Which player from the Lakers had the best shooting percentage
- Which player from the Lakers had the worst shooting percentage
- Same items for Nuggets
- Which players scored the most points, etc.

5th grade Kara's class

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You can't make this stuff up

- Gettysburg Outlets – July 3, 2009. 50% off sale on all purchases at the Izod store. Sign indicates 50% off the all-store sale.
  - Patron – "well that means it's free."
  - Clerk – "no sir, it's 50% off the 50% off sale."
  - Patron – "well, 50% + 50% is 100% so that means it should be free."
- This went on for a while. AND, there was a sign indicating 70% off for some items, meaning 70% off the 50% off original sale, which our patron would interpret as the item being free and 20% in cash!

Percent Benchmarks

0%

100% **50%** **< 10%**

~25% **~75%** **~90%**

> 50% **< 50%**

- Lefthanders in the room or class
- Once lived in **New Jersey**
- Been involved in education > 10 years
- People who were born in **Louisiana**

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Concluding Thoughts

- We need to develop, maintain, and brag about effective RtI programs in mathematics.
- Kids can do the important math they must know to continue with this subject and we must find ways to ensure this…
- The Focal Points, Critical Foundations, and CCSS Domains provide essential building blocks for ALL students.
- Number sense, while elusive, must be nurtured – every day!
- A sense of number breeds confidence.
- Don't forget, fractions – all of 'em - are numbers too!